Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Topology

Subject Code: 5SC01TOP1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date:28/03/2017** **Time:10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define:Topological space. **(01)**

b. Define: Projection map. **(01)**

c. Define: Compact space. **(01)**

d. Define: Locally compact space. **(01)**

e. Define: Interior of set. **(01)**

f. Give an example of T_1 space which is not T_2. **(02)**

Q-2 **Attempt all questions** **(14)**

a) Define closure of a set. Let A be a subset of topological space X and $A^{'}$ be the set of all limit points of A. Then prove that $\bar{A} = A \cup A^{'}$. **(05)**

b) State and prove sequence lemma. **(05)**

c) State and prove pasting Lemma. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) Let $\mathcal{B}$ and $\mathcal{B}^{'}$ be bases for the topology τ and $\tau^{'}$ respectively on X. Then prove that the following are equivalent: **(05)**
(i) $\tau^{'}$ is finer than τ
(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x there is a basis element $B^{'} \in \mathcal{B}^{'}$ such that $x \in B^{'} \subset B$.

b) If $\mathcal{B}$ is a basis for the topology of X then prove that the collection $\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$ is a basis for the subspace topology on Y. **(05)**

c) Let Y be a subspace of X. Let A be a subset of Y and $\bar{A}$ denote the closure of A in X. Then prove that the closure of A in Y is $\bar{A} \cap Y$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Let X and Y be topological spaces and $f: X \rightarrow Y$ then prove that following are equivalent **(08)**
(i) f is continuous.
(ii) For every subset A of $X, f(\bar{A}) \subseteq \overline{f(A)}$.
(iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.
(iv) For each $x \in X$& each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

b) If (X, τ) be a topological space and $A, B \subset X$ then prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$. Is it true $\overline{A \cap B} = \bar{A} \cap \bar{B}$? Justify your answer. **(04)**

c) State finite intersection property. **(02)**

OR

Q-3 **a)** Let $f: A \rightarrow X \times Y$ be a given by the equation $f(a) = (f_1(a), f_2(a))$. Then prove that f is continuous if and only if the functions $f_1: A \rightarrow X$ and $f_2: A \rightarrow Y$ are continuous. **(06)**

b) Let $f : X \rightarrow Y$. If the function f is continuous then prove that for every convergent sequence $x_n \rightarrow x$in X, the sequence $f(x_n)$ converges to $f(x)$. The converse holds if X is metrizable. **(05)**

c) Let X, Y, Z be topological spaces. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are continuous functions then prove that $gof: X \rightarrow Z$ is continuous. **(03)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Is discrete topological space a T_1 space? **(01)**

b. Define: Homeomorphism. **(01)**

c. Define: Normal space **(01)**

d. Define: Separable space. **(01)**

e. Define: Disconnected topological space. **(01)**

f. State Tychonoff theorem. **(02)**

Q-5 **Attempt all questions** **(14)**

a) Prove that every closed subspace of a compact space is compact. **(05)**

b) Prove that every metrizable space is normal **(05)**

c) Give an example of compact space which is not Hausdorff.. **(04)**

OR

Q-5 **a)** Prove that the image of a connected space under a continuous map is connected. **(05)**

b) Prove that every compact subspace of T_2 space is closed. **(05)**

c) Show that every compact subspace of a metric space is bounded. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Prove that closed subspace of a locally compact space is locally compact. **(05)**

b) Prove that every compact T_2 space is normal. **(05)**

c) Prove that the continuous image of a sequentially compact set is sequentially compact. **(04)**

OR

Q-6 a) State and prove Urysohn's Lemma **(14)**